Verification of Sequential Convex Programming for Parametric Non-convex Optimization

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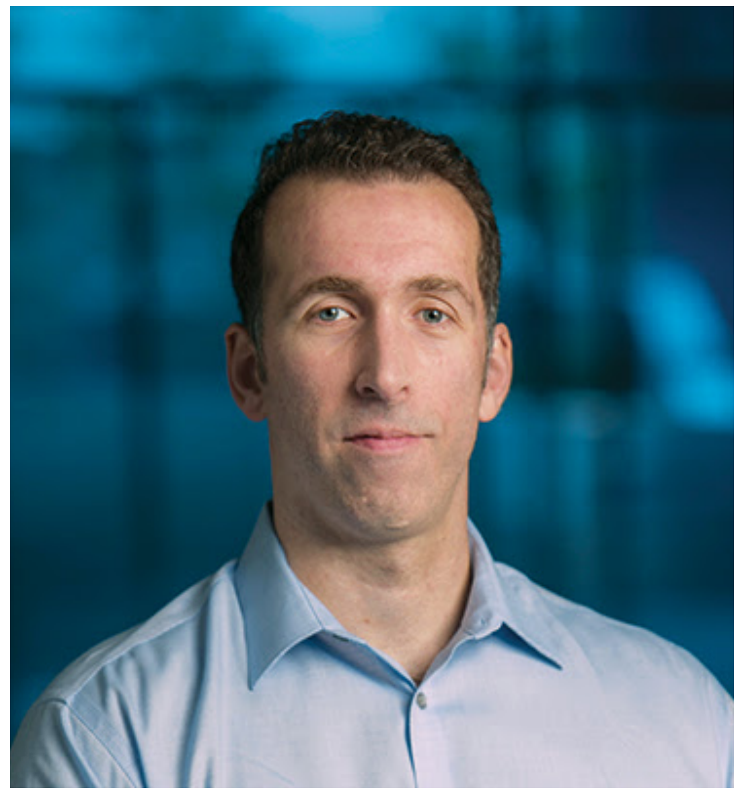


Nikolai Matni

George Pappas

Real-world optimization is parametric

parameter

$x \longrightarrow$

$$\begin{array}{ll} \text{minimize} & f(z, x) \\ \text{subject to} & g(z, x) \leq 0 \end{array}$$

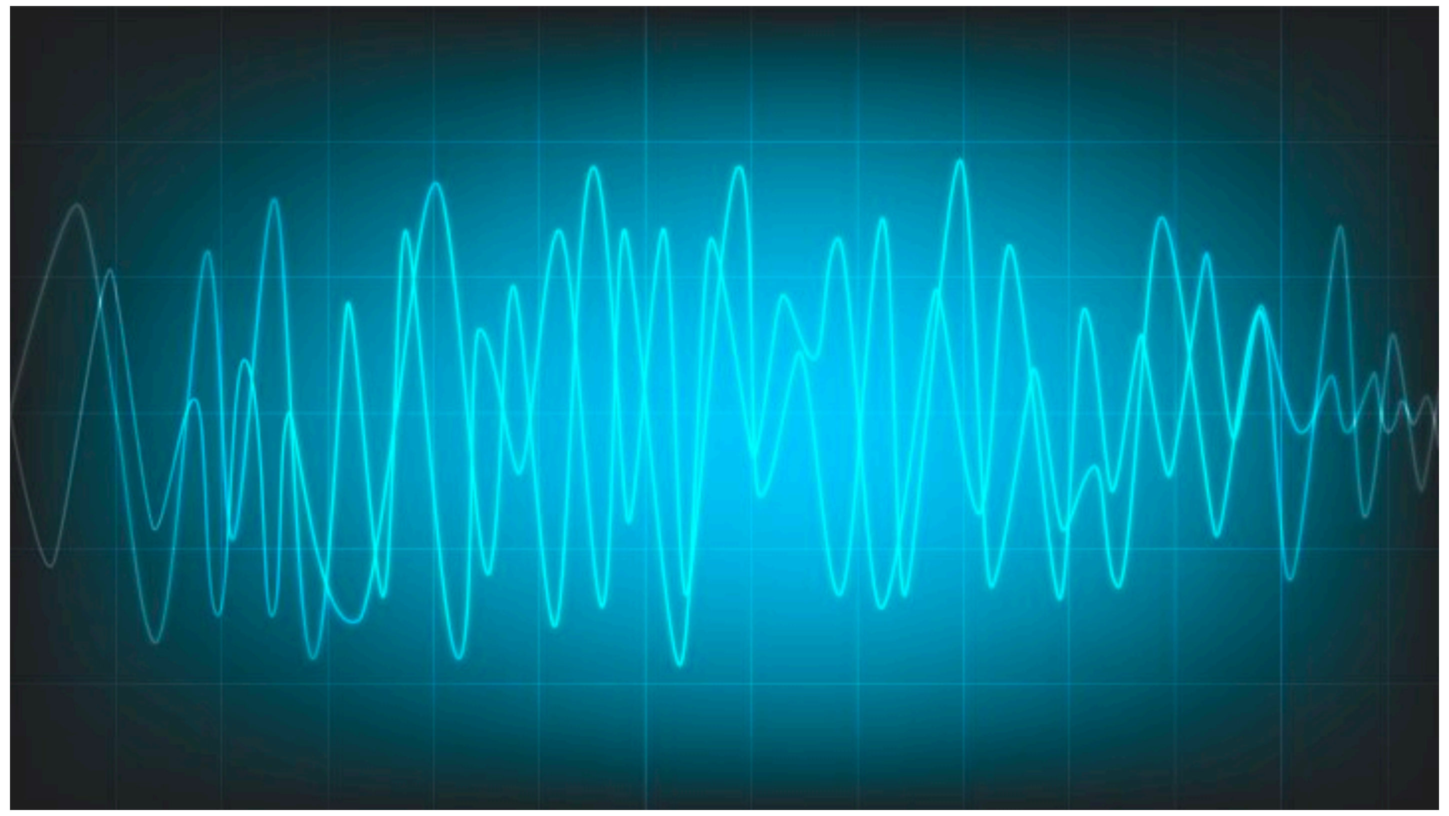
optimal solution

$\longrightarrow z^{\star}(x)$

robotics and control

energy

signal processing



most applications require **fast decisions in real-time**

Sequential convex programming is popular for real-time optimization

parameter
$x \longrightarrow$

minimize $\quad f(z, x)$
subject to $\quad g(z, x) \leq 0$

f and g non-convex in z

optimal solution
$\longrightarrow z^\star(x)$

global methods (slow)

- branch and bound
  - global certificate of optimality
  - too slow for real-time apps

Lawler and Wood 66
Belotti et al. 13

local methods (fast)

- sequential convex programming
  - lack of global guarantees
  - involves solving convex problems

Nocedal and Wright, 06
Boyd et al., 08

our focus in this talk

Sequential Convex Programming (SCP) methods

sequential convex programming

initialize $z^0(x) = 0$

solve convex problem $z^{k+1}(x) \in T(z^k(x), x)$

for $k = 0, \ldots, \boxed{K} - 1$ steps

budget of iterations

a zoo of SCP methods

- trust-region method — Nocedal and Wright, 06
- prox-linear method — Drusvyatskyi and Paquette, 19
- convex-concave procedure — Lipp et al., 16
- relax-round-polish method — Diamond et al., 17

parameter x

current iterate $z^k(x)$

$$
\begin{array}{ll}
\text{minimize} & \hat{f}(z, x, z^k(x)) \\
\text{subject to} & \hat{g}(z, x, z^k(x)) \leq 0 \\
& \|z - z^k(x)\|_\infty \leq \rho
\end{array}
$$

next iterate $z^{k+1}(x)$

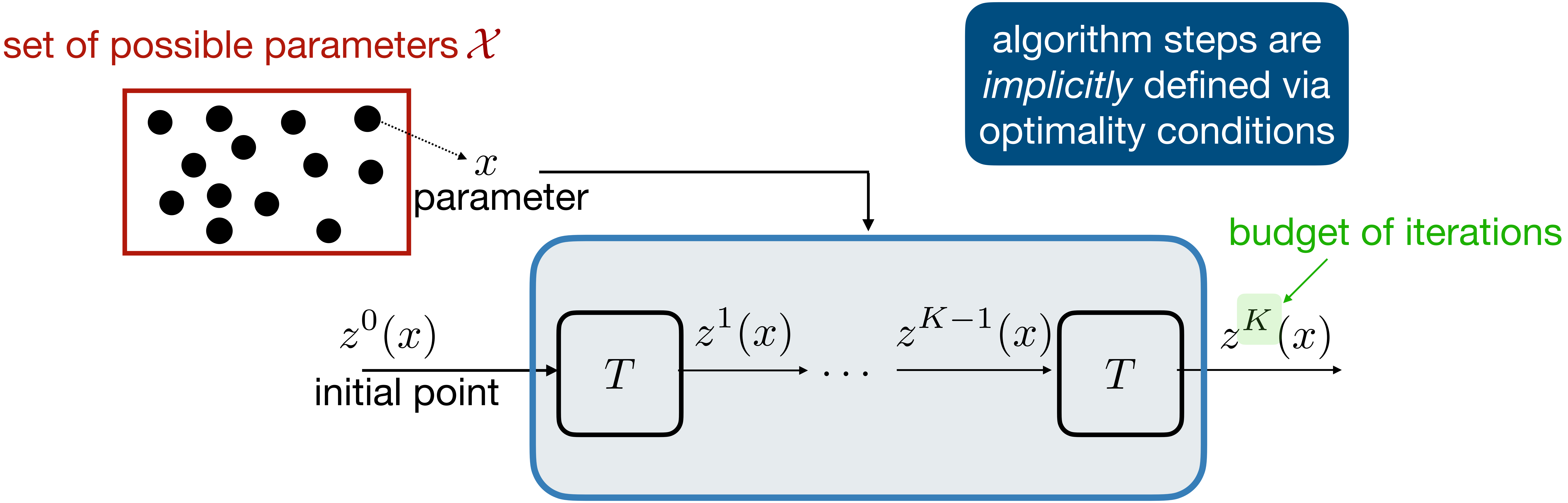
trust-region size

local convex quadratic approximations around $z^k(x)$

performance depends on

- initial point $z^0(x)$
- trust region size ρ
- parameter x

SCP as implicit fixed-length computational graphs



our question: can we obtain worst-case guarantees of sequential convex programming algorithms for all parameters in a given set $\mathcal{X}$?

Existing literature and main idea

our question: can we obtain worst-case guarantees of sequential convex programming algorithms for all parameters in a given set $\mathcal{X}$?

existing guarantees:
convergence to stationary point

guaranteed under certain assumptions

Drusvyatskiy and Paquette 2019; Bonalli et al. 2019;
Roulet et al. 2025; Beck 2017; Nocedal and Wright 2006

limitation: no global guarantee on worst-case performance

- don't take advantage of parametric structure

main idea:

- **parametric** structure can be exploited!

how can we derive worst-case guarantees for $x \in \mathcal{X}$?

- suboptimality $\longleftarrow$ our focus next
- level of constraint violation
- feasibility of subproblems

Main idea for verifying suboptimality

original non-convex problem

$$\begin{array}{ll} \text{minimize} & f(z, x) \\ \text{subject to} & g(z, x) \leq 0 \end{array}$$

assumption: constraints always satisfied

$$g(z^k(x), x) \leq 0$$

(*e.g.*, unconstrained or convex constraints)

---

search for the worst-case parameter with a "meta"-optimization problem!

how to encode the SCP convex opt problems?

$$\begin{array}{lll} \text{maximize} & \text{(performance metric)} & f(z^K, x) - f(z^\star, x) \\ \text{subject to} & \text{(algorithm steps)} & z^{k+1} \in T(z^k, x) \quad k = 0, \dots, K-1 \\ & \text{(parameter)} & x \in \mathcal{X} \\ & \text{(initial point)} & z^0 \in S \\ & \text{(optimality)} & z^\star \in \arg\min_z \ f(z, x) \quad \text{s.t.} \quad g(z, x) \leq 0 \end{array}$$

e.g., $S = \{0\}$, $S = \{z^{\text{ws}}\}$

how to encode this optimality constraint?

KKT conditions of convex quadratic programs

our convex subproblems are convex QPs

Primal

$$\begin{array}{ll} \text{minimize} & (1/2)u^T P u + c^T u \\ \text{subject to} & Au + s = b \\ & s \geq 0 \end{array}$$

Dual

$$\begin{array}{ll} \text{maximize} & -(1/2)u^T P u - b^T y \\ \text{subject to} & Pu + A^T y + c = 0 \\ & y \geq 0 \end{array}$$

KKT conditions

$$Au + s = b, \quad Pu + A^T y + c = 0, \quad s \geq 0, \quad y \geq 0, \quad s^T y = 0$$

KKT are necessary and sufficient

Boyd and Vandenberghe 04

flatten SCP iterations via KKT constraints in the verification problem

The suboptimality verification problem

original non-convex problem

$$\begin{array}{ll} \text{minimize} & f(z, x) \\ \text{subject to} & g(z, x) \leq 0 \end{array}$$

assumption: constraints always satisfied

$$g(z^k(x), x) \leq 0$$

(*e.g.*, unconstrained or convex constraints)

$$\begin{array}{lll} \text{maximize} & \text{(performance metric)} & f(z^K, x) - f(z^\star, x) \\ \text{subject to} & \text{(algorithm steps)} & z^{k+1} \in T(z^k, x) \quad k = 0, \dots, K-1 \\ & \text{(parameter)} & x \in \mathcal{X} \\ & \text{(initial point)} & z^0 \in S \\ & \cancel{\text{(optimality)}} & \cancel{z^\star \in \arg\min_z \; f(z, x) \quad \text{s.t.} \quad g(z, x) \leq 0} \\ & \text{(feasibility)} & g(z^\star, x) \leq 0 \end{array}$$

encode the KKT conditions

relax with a feasibility constraint

this relaxation is tight!

verification done offline (time-insensitive setting)

…to enable reliable optimization online (time-sensitive setting)

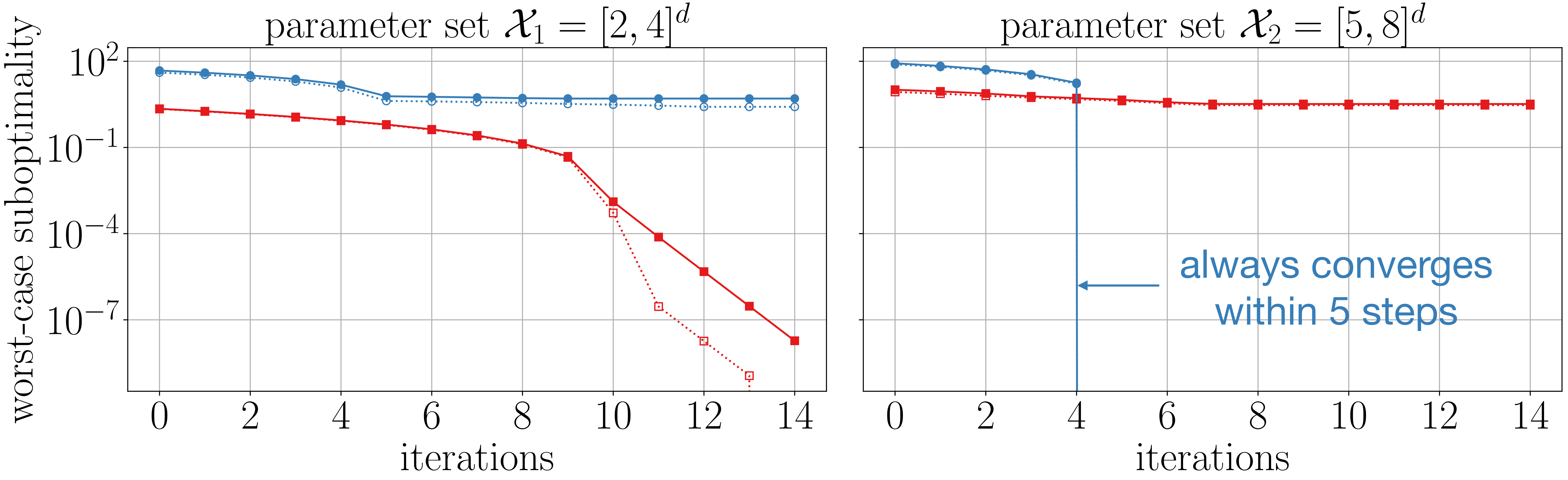
Non-convex quadratic optimization over a box

$P \not\succeq 0$

parameter x →

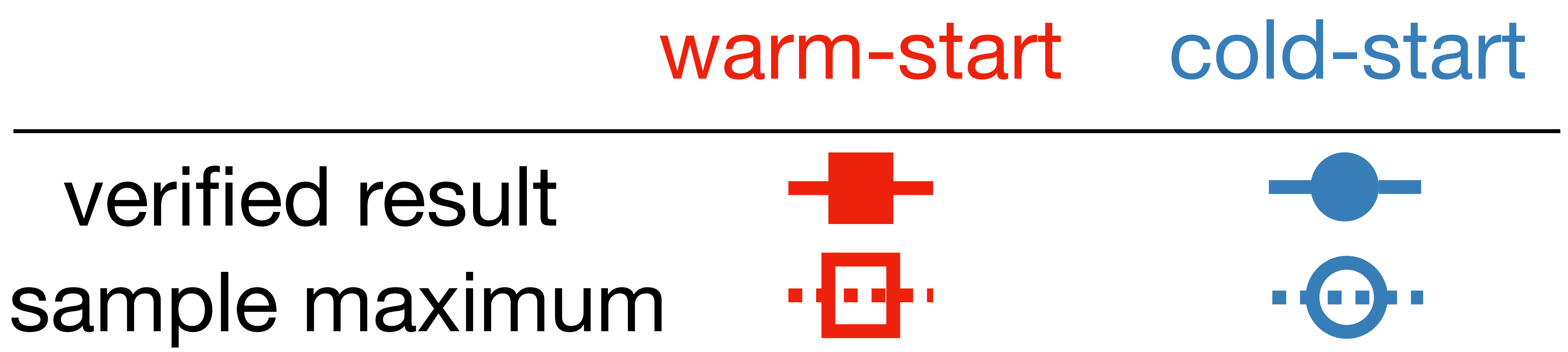
$$\begin{array}{ll} \text{minimize} & (1/2) z^T P z + x^T z \\ \text{subject to} & -1 \leq z \leq 1 \end{array}$$

→ solution $z^\star(x)$



can certify global optimality (or not)!

initial point, trust region size, parameter set matter



conventional wisdom (warm-start better than cold-start) not always right!

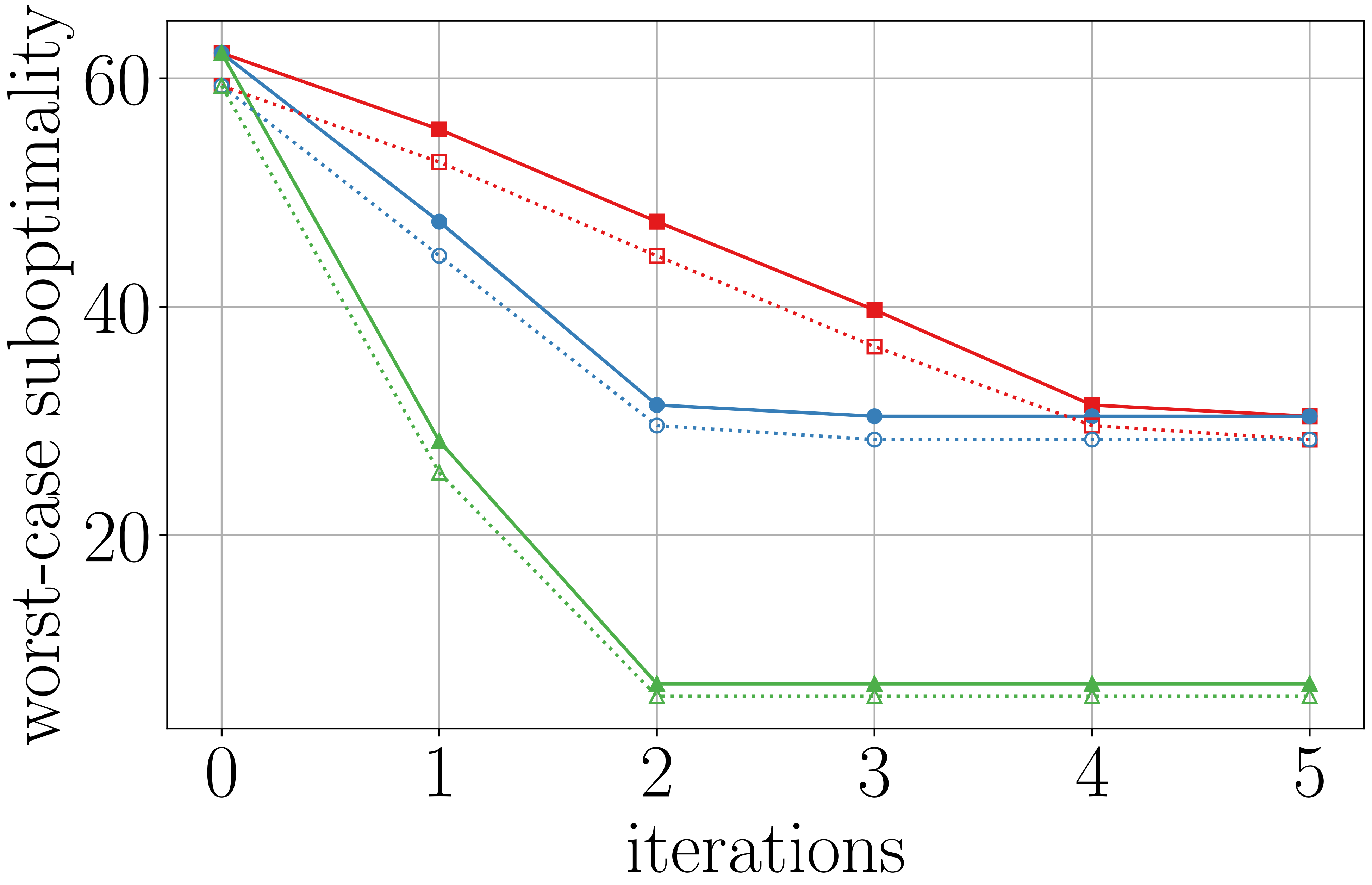
Network utility maximization

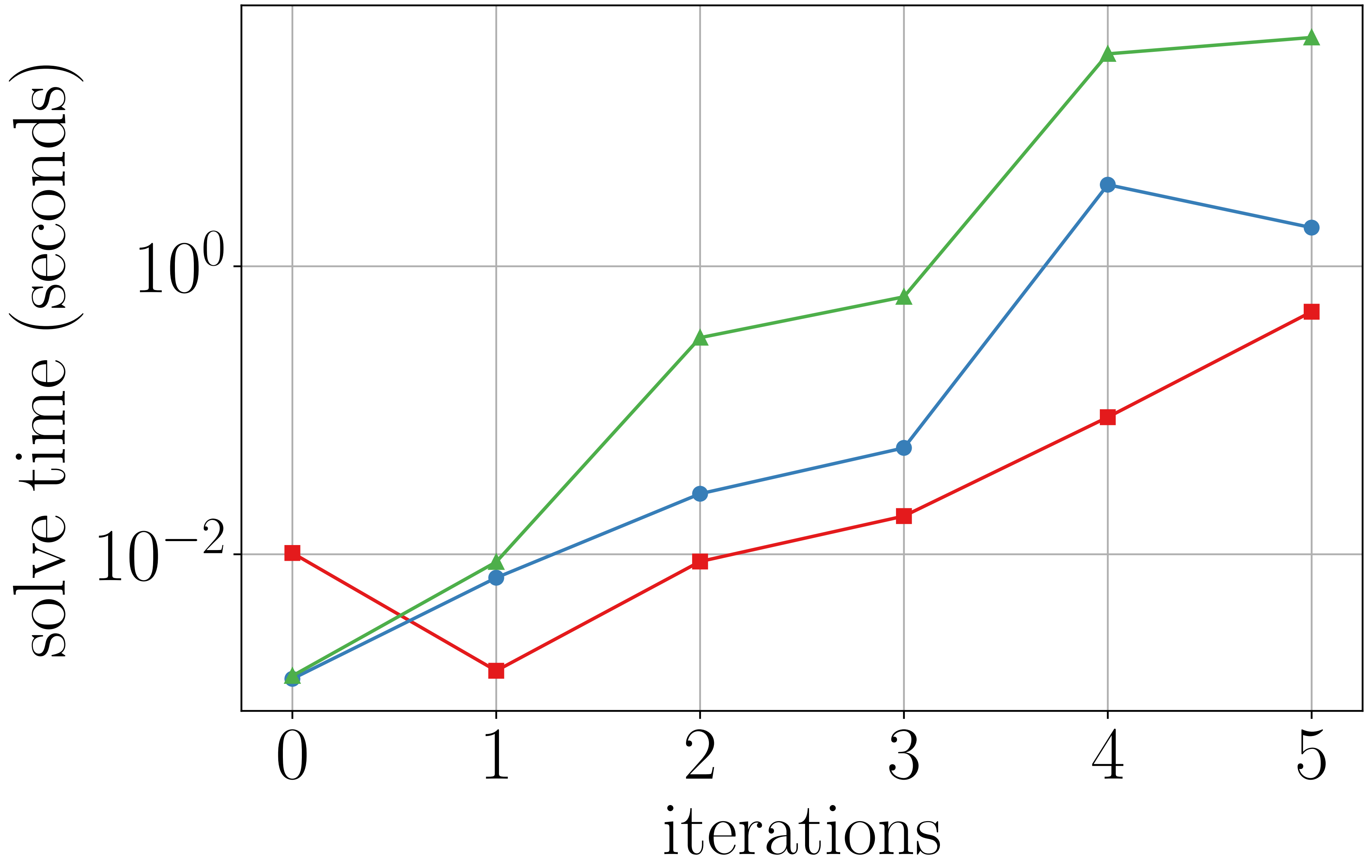
Guisewite and Pardalos, 90 Fazel and Chang, 05

capacities x →

$$\begin{array}{ll} \text{maximize} & (1/2)\|z\|_2^2 \\ \text{subject to} & Rz \le x, \quad z \ge 0 \end{array}$$

→ flows $z^\star(x)$





trust-region size ρ

	$\rho = 1$	$\rho = 2$	$\rho = 5$
verified result			
sample maximum			

the largest trust-region size performs best

verification time scales with iterations

Handling relaxing and rounding schemes

what about relaxing and rounding schemes?

relax-round-polish

Diamond et al. 17

step 1: solve a convex **relaxation**

step 2: **round** (non-convex projection)

step 3: **polish** the continuous variables

example

$$\begin{array}{ll} \min & (1/2)\|Az - x\|_2^2 \\ \text{s.t.} & \|z\|_0 \leq k \end{array}$$

$$\min \quad (1/2)\|Az - x\|_2^2 + \lambda \|z\|_1$$

project onto $\{z \mid \|z\|_0 \leq k\}$

solve unconstrained least squares w/ fixed support

relax-round-polish generalizes convex relaxations

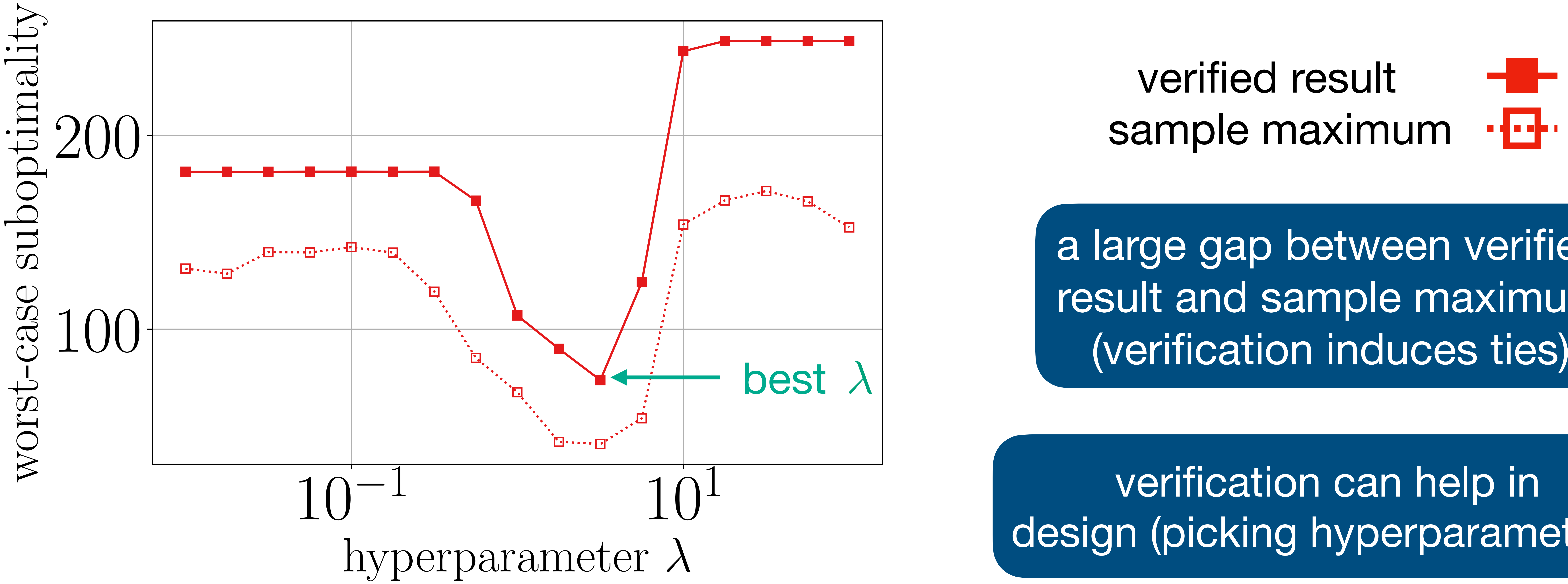
all steps can be included in the verification problem!

Sparse coding

noisy measurement x → **signal reconstruction**

$$\begin{array}{ll} \text{minimize} & (1/2)\|Az - x\|_2^2 \\ \text{subject to} & \|z\|_0 \leq k \end{array}$$

→ reconstructed signal $z^\star(x)$



a large gap between verified result and sample maximum (verification induces ties)

verification can help in design (picking hyperparameters)

Verifying feasibility of subproblems

what if a subproblem is infeasible?

relax-round-polish

Diamond et al. 17

step 1: solve a convex **relax**ation

step 2: **round** (non-convex projection)

step 3: **polish** the continuous variables

example

$$\begin{array}{ll} \min & w^T P w + c^T w \\ \text{s.t.} & Aw + Bv \leq d, v \in \{0,1\}^{d_v} \end{array}$$

relax binary vars and solve LP

$\bar{v} =$ round binary vars

polish continuous vars

$$\begin{array}{ll} \min & w^T P w + c^T w \\ \text{s.t.} & Aw + Bv \leq d, v = \bar{v} \end{array}$$

may not be feasible!

feasibility detection: use the Farkas lemma in the verification problem!

verification solution gives a feasibility or infeasibility certificate

Hybrid vehicle control

load

P_t^{load}

$$
\begin{array}{ll}
\text{minimize} & (E_T - E^{\max})^2 + \sum_{t=0}^{T-1} \alpha (P_t^{\text{eng}})^2 \\
\text{subject to} & E_{t+1} = E_t - \tau P_t^{\text{batt}} \\
& P_t^{\text{batt}} + P_t^{\text{eng}} = P_t^{\text{load}} \\
& E^{\min} \leq E_t \leq E^{\max} \\
& 0 \leq P_t^{\text{eng}} \leq z_t P^{\max} \\
& z_t \in \{0, 1\}
\end{array}
$$

engine use

P_t^{eng}

battery use

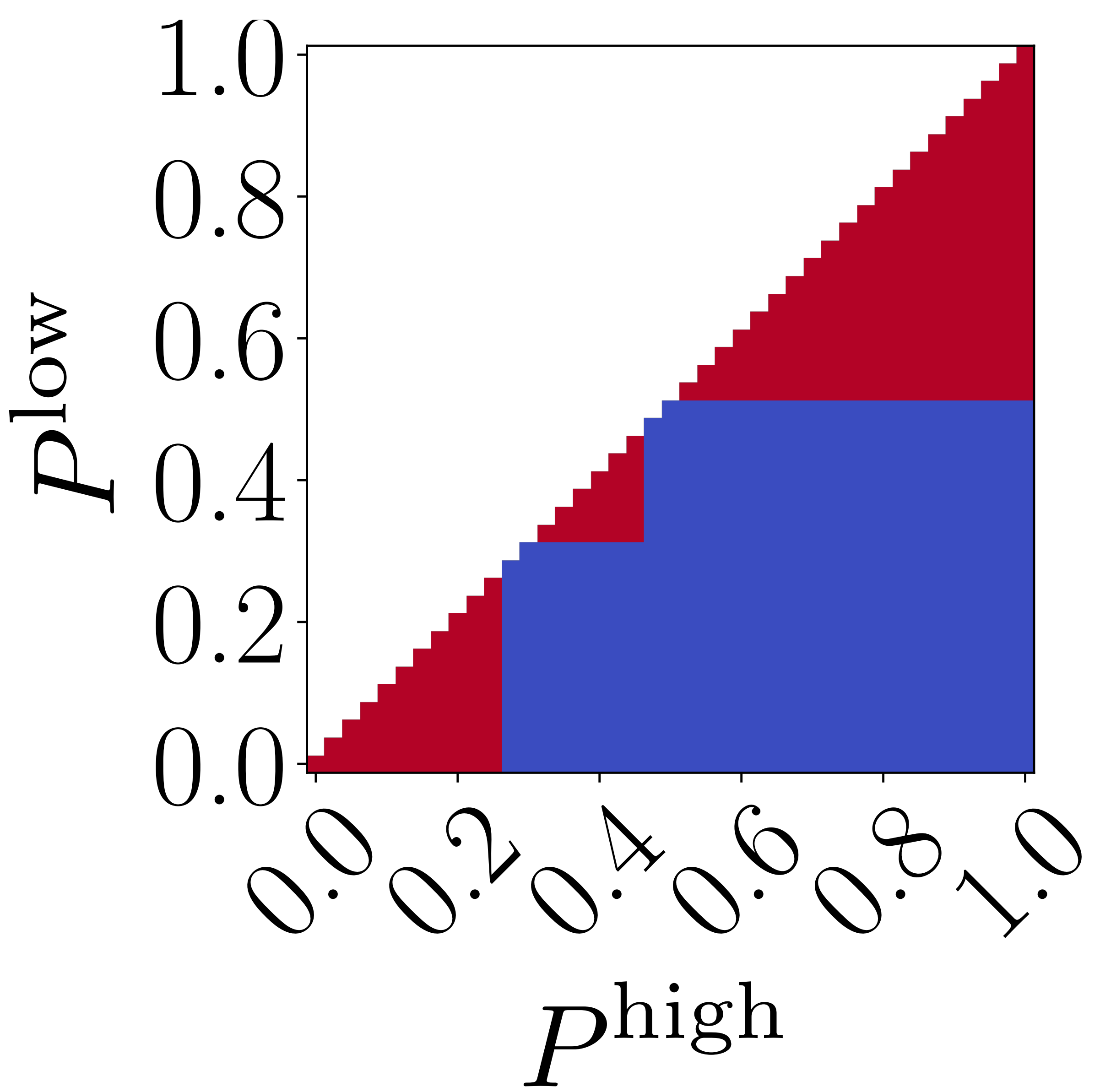
$P_t^{\text{batt}} \quad E_t$

parameter set $\mathcal{X}$

$P^{\text{low}} \leq P_t^{\text{load}} \leq P^{\text{high}}$

feasibility verified

infeasibility verified



for some parameter ranges, we can verify feasibility of the polish step

Connection to neural network verification

	neural network verification Tjeng et al. 17 Fazlyab et al. 19, 22 Ceccon et al. 22 Albarghouthi 21 Anderon et al. 20 Wang et al. 21	**sequential convex programming verification** this work
input		problem instance with parameter x
uncertainty set	base image deviation $\mathcal{X} = \{\bar{x} \mid \|\bar{x} - x_0\|_\infty \leq \delta\}$	set of possible parameters $\mathcal{X}$
computation steps	linear + ReLU steps	algorithm steps
worst-case guarantee	predicted class (panda) doesn't change for any perturbation	worst-case performance

Conclusion

- **parametric** structure can be exploited
- verification obtains **exact global worst-case guarantees**
  - suboptimality, constraint violation, subproblem feasibility
- applies to **many SCP algorithms**, relaxing+rounding schemes

traditional function-class view

- pen-and-paper analysis
- one-size fits all
- limited (local) guarantees

→

new parametric view

- computer-assisted analysis
- family-specific
- exact global guarantees

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arXiv arxiv.org/pdf/2511.10622

github.com/rajivsambharya/verify_scp

rajivsambharya.github.io